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Dynamics of a disabled population in Morocco

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Abstract

Background: The disabled population constitutes a class of people needing special care and necessitating important economic and social effort.

Methods: In this paper, using specific parameter settings, partial differential equations are used to model the temporal change of the proportion of the disabled population in Morocco.

Results: Combining different forms and values of the parameters, a numerical method is proposed and three scenarios are considered. These forms and values are determined by data fitting and simulation.

Conclusions: The experiments show clearly the dynamical evolution of the disabled population with time and age according to each scenario.

Introduction

Although the definition of disability may vary from one country to another, the handicapped population represents a special category which appeals for specific needs and more attention. Around the world, it is estimated that more than 10% of the population can be classified as disabled. However, the numbers given by different countries vary. In 1987, 32 million people of the United States were considered disabled, accounting for 13.5% of the total population [1]. A survey in the United Kingdom indicated in 1988 that 6.5 million people representing 14% of the population, could be disabled [2]. In Morocco, 10% of the population suffer from physical disability [3], whereas in China nearly 52 million, constituting 5% of the whole population were considered disabled according to a sampling survey carried out in 1988 [4]. Zhou and Li [5] considered a matrix model of the disabled population with application to Chinese data. Boutayeb and Derouich [6] stressed the effect of a natural " accumulation" with age in chronical diseases.

In this paper we consider the disabled continuous model with age structure. The dynamics are modelled by hyperbolic partial differential equations with initial condition and a nonlocal boundary condition at the zero age. The presence of integral in the boundary condition is well known, accounting for the new born from the population. Numerical methods are proposed and simulation is carried out with different values of the parameters. Throughout this paper the terms disabled and handicapped are used interchangeably to refer to the inability or incapacity to meet certain standard, physical, social, occupational or economical responsibility.

Formulation of the model and numerical method

Notations

In our model, the total population is divided into two groups: the healthy and the disabled. The following notations will be used throughout this paper.

$x(a, t)$: the number of the healthy population aged a at time t,

$y(a,t)$: the number of the disabled population aged a at time t,

$u(a,t) = x(a,t) + y(a,t)$: the number of the total population aged a at time t,

$s(a,t)$: the survival rate of healthy population aged a at time t,

$d(a,t)$: the death rate of healthy population aged a at time t,

$d'(a,t)$: the death rate of disabled population aged a at time t, ($d' = d + \delta$),

$q(a,t)$: the survival rate of disabled population aged a at time t,

$\beta(a, t)$: the fertility rate of the healthy women aged a at time t,

$\gamma(a, t)$: the fertility rate of the disabled women aged a at time t,

$r(a,t) = \dfrac{y(a,t)}{u(a,t)} \times 100\%$: the percentage of the disabled population aged a at time t,

$c(a,t)$: the handicapping rate of the healthy population aged a at time t,

$e(a,t)$: the rehabilitating rate of the disabled population aged a at time t,

A : final age.

Formulation of the model

The handicapping rate $c(0, t)$ is assumed to be the percentage of the congenital disability rate in year t. Assuming that the number of males is equal to the number of females, from these assumptions and population theory, we get the following model

$$\begin{cases} \dfrac{\partial x}{\partial t}(a,t) + \dfrac{\partial x}{\partial a}(a,t) = -c(a,t)s(a,t)x(a,t) + e(a,t)q(a,t)y(a,t) - d(a,t)x(a,t), & (a,t) \in [0,A] \times J, \\ \dfrac{\partial y}{\partial t}(a,t) + \dfrac{\partial y}{\partial a}(a,t) = c(a,t)s(a,t)x(a,t) - e(a,t)q(a,t)y(a,t) - d'(a,t)y(a,t), & (a,t) \in [0,A] \times J, \\ x(0,t) = \dfrac{1-c(0,t)}{2}\displaystyle\int_0^A \left(\beta(a,t)x(a,t) + \gamma(a,t)y(a,t)\right)da, & t \in J, \\ y(0,t) = \dfrac{c(0,t)}{2}\displaystyle\int_0^A \left(\beta(a,t)x(a,t) + \gamma(a,t)y(a,t)\right)da, & t \in J, \\ x(a,0) = x_0(a), y(a,0) = y_0(a), \; a \in [0,A]. \end{cases}$$

Where $\beta(a,t) = 0$, $\gamma(a,t) = 0$, when $a < A_0$ or $a > A_1$ and $[A_0, A_1]$ is the child bearing age of women and J is the time interval $[0,T]$.

By adding $x(a,t)$ and $y(a,t)$ and using the notations in subsection (2.1) we obtain :

$$\begin{cases} \dfrac{\partial u}{\partial t}(a,t) + \dfrac{\partial u}{\partial a}(a,t) = -m(a,t)u(a,t), & (a,t) \in [0,A] \times J, \\ \dfrac{\partial r}{\partial a}(a,t) + \dfrac{\partial r}{\partial t}(a,t) = \delta(a,t)r^2 - \varepsilon(a,t)r + \omega(a,t), & (a,t) \in [0,A] \times J, \\ u(0,t) = \displaystyle\int_0^A b(a,t)u(a,t)da, \; t \in J, \\ u(a,0) = u_0(a), \; a \in [0,A], \end{cases} \tag{1}$$

where $m(a, t) = d(a,t) + \delta(a,t)r(a,t)$, $b(a,t) = \frac{1}{2}(\beta(a, t)(1 - r(a,t)) + \gamma(a, t))$, and $\varepsilon(a, t) = (c(a, t)s(a, t) + e(a, t)q(a, t) + \delta(a, t))$, $\omega(a, t) = c(a,t)s(a,t)$. This leads to the following equation

$$\begin{cases} \dfrac{\partial r}{\partial a}(a,t) + \dfrac{\partial r}{\partial t}(a,t) = \delta(a,t)r^2(a,t) - \varepsilon(a,t)r(a,t) + \omega(a,t), \\ r(0,t) = c(0,t), \; t \in J, \\ r(a,0) = r_0(a), \; a \in [0,A]. \end{cases} \tag{2}$$

Numerical approximation

The partial differential equation (2) is solved over a mesh of T years time and A years of age. The time and the age variables will be discretized, respectively, at the points $t_n = nl$, $n = 0,...,N$ and $a_m = mh$, $m = 0,...,M$.

Hence, if a final age A is considered and the usual age categories of ten years observed every five years on a century scale then $A = Mh$ and $T = Nl$ so that for $A = T = 100$, $l = 5$ years and $h = 10$ years.

Discretizing equation (2) in time and age and using the Lax-Wendroff method [7] gives:

$$r_m^{n+1} = \left(1 - p^2\right)r_m^n - \frac{1}{2}p(1-p)r_{m+1}^n + \frac{1}{2}p(1+p)r_{m-1}^n + l\delta_m^n\left(r_m^n\right)^2 - l\varepsilon_m^n r_m^{n+1} + l\omega_m^n,$$

where $p = \dfrac{l}{h}$, this yields the explicit scheme

$$r_m^{n+1} = \frac{\left(1 - p^2 + l\delta_m^n r_m^n\right)r_m^n - \frac{1}{2}p(1-p)r_{m+1}^n + \frac{1}{2}p(1+p)r_{m-1}^n + l\omega_m^n}{1 + l\varepsilon_m^n} \quad n = 0,\cdots,N-1, m = 1,\cdots,M. \tag{3}$$

A standard von Neumann stability analysis [7] shows that this numerical scheme is stable and convergent provided that $p \leq 1$.

The literature on theoretical and numerical solutions of structured population models is abundant [3,8,9]. In search of clarity and simplicity, we considered the percentage $r(a,t)$ of the disabled population as given by equation (2) and easily implemented by the numerical scheme (3).

Parameters expressions

The general trends suggested for the parameters are basically based on two sources [10,11]: the 1971, 1982 and 1994 national censuses of population in Morocco, and regional surveys related to health and growth of population based on age structure. When sufficient data exist for various values of age and time, parameters are determined by fitting continuous curves to discrete data. Otherwise, explicit forms of the parameters are proposed on the basis of simulations, assumptions, and comparisons with the United Nations and World Bank long-range world population projections [12].

1. Natural death rate of healthy population : $d(a,t)$

With 50% of the population under the age of 20 and 90% less than 55 years old, the Moroccan population is young. Notwithstanding a decreasing tendency in fertility rate and an improvement of life expectancy, the age structure is not expected to change rapidly. It is assumed that this rate is linearly decreasing with age and slowly decreasing with time. The following explicit form is used

$$d(a,t) = \frac{T}{T+t} \times (c_1 a - c_2),$$

2. Death rate of disabled population : $d'(a, t)$

As stressed by many studies [3], this rate differs from the death rate of the healthy population by $\delta(a,t)$ so that

$$d'(a,t) = d(a,t) + \delta(a,t),$$

where the expression of $\delta(a, t)$ is assumed to be given by

$$\delta(a, t) = c_3 + c_4 a^2 - c_5 t$$

(quadratic form for medium increase with age and decrease with time).

3. Rehabilitating rate : $e(a, t)$

Some disabilities may be cured, provided they are diagnosed at the early stage. It is assumed that $e(a, t)$ has a form which is slowly increasing with time and decreasing with age, namely:

$$e(a,t) = c_6 \times \frac{t}{T+t} \times \frac{A}{A+a}.$$

4. Handicapping rate : $c(a, t)$

As mentioned by the authors [3], several forms for the handicapping rate can be suggested. Throughout this paper three forms will be used for $c(a, t)$:

- Linear form for slow increase with age and decrease with time

$$c(a, t) = c_7 + c_8 a - c_9 t.$$

- Quadratic form for medium increase with age and decrease with time

$$c(a,t) = c_{10} \times a \times (c_{11} - a) - c_{12} \times t.$$

- Exponential form for rapid increase with age and decrease with time

$$c(a,t) = (c_{13} \times a \times (c_{14} - a) - c_{15} t)\ e^{c_{16} a}.$$

The parameters $c_1,\ldots,c_5$ were determined by fitting the continuous curves $d(a,t)$ and $\delta(a,t)$ to discrete data. Whereas the parameters $c_6,\ldots,c_{16}$ were determined by simulations based on possible scenarios.

Values of the parameters used are given in table 1.

Table 1: Parameters of the model

parameter	c_1	c_2	c_3	c_4	c_5	c_6	c_8	c_8
value	0.009	0.001	0.00001	0.00001	0.001	0.00005	0.00004	0.00001
parameter	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}
value	0.001	0.00001	0.6	0.0001	0.0001	0.6	0.00001	0.00001

Table 2: Temporal change of the disabled population with scenario (1)

time	5	10	15	20	25	30	35	40	45	50
age 10	2.44	2.7	2.9	3.14	3.9	4.29	4.53	4.87	5.68	6.17
age 20	2.62	3.17	3.54	4.22	5.19	6.77	7.18	8.26	9.45	7.31
age 30	2.84	4.51	5.01	5.54	6.21	7.33	8.81	8.20	9.46	7.33
age 40	2.98	5.32	6.71	7.25	8.75	9.13	9.87	10.21	14.43	12.67
age 50	3.19	6.43	7.12	8.44	9.03	10.15	11.24	12.37	13.76	14.34
age 60	3.45	7.21	8.18	9.51	10.33	11.7	12.89	15.51	16.96	20.34
age 70	3.69	8.75	10.5	11.7	12.21	13.56	14.84	16.58	21.36	25.67

Table 3: Temporal change of the disabled population with scenario (2)

time	5	10	15	20	25	30	35	40	45	50
age 10	2.9	3.06	4.17	5.21	6.68	8.41	10.81	11.5	11.87	12.28
age 20	2.95	3.43	4.48	5.51	6.72	9.21	10.71	12.27	13.53	13.79
age 30	3.05	5.12	4.91	6.19	7.33	10.08	11.81	13.74	14.31	15.10
age 40	3.75	4.23	5.11	7.88	8.44	12.65	13.71	14.23	15.77	16.48
age 50	3.98	5.71	7.33	9.12	10.14	13.42	14.98	17.23	19.13	20.74
age 60	4.08	8.41	9.8	11.38	12.51	14.72	16.15	18.25	20.38	22.56
age 70	4.25	11.33	13.15	16.13	19.11	22.42	23.55	25.75	28.38	29.86

Results

Several scenarios may be considered by combining different forms and values of the parameters to simulate the dynamics of the handicapped population between age 0 and 100 years in a certain time interval. In the present study, we concentrate on three scenarios. Scenario (1) low congenital disability with linear form of handicapping rate. Scenario (2) medium congenital disability with quadratic form of handicapping rate. Scenario (3) high congenital disability with exponential form of handicapping rate. The numerical method (3) was implemented with different values of the parameters. The output gives the percentages of the disabled population for a range of time and age.

In the first scenario, a low congenital disability (c_0 = 0.001) and a slow handicapping rate (linear form of $r(a,t)$) were used to calculate the percentage of the disabled population shown in table 2. Similarly, a medium congenital disability (c_0 = 0.005) and a medium form of the handicapping rate (quadratic form of $r(a,t)$) were used, with the results given in table 3. Finally, a high congenital disability (c_0 = 0.01) and high handicapping rate (exponential form of $r(a, t)$) yielded the percentages of the disabled population given in table 4.

Discussion

Disability results from a combination of biomedical, demographic and personal factors, interacting with environmental and social conditions. However, the incidence and prevalence in a particular environment may be controlled at at least three levels.

1. It is well known that some chronic diseases significantly contribute to the number of disabled people. For instance, diabetes is the commonest cause of blindness in people under the age of 65 in the UK, and has been reported to account for 46% of lower-limb amputations carried by the NHS [13,14]. Similar figures are reported in Europe and the United States, where the total annual economic cost of diabetes in 1997 was estimated to be US $98 billion of which US $54 billion for indirect costs attributed to disability and mortality. In Morocco and developing countries in general, the lack of medical care makes the situation even worse. Instead of struggling to find a way to look after people with blindness, amputations or kidney failure, why not try to avoid or at least to delay as much as possible the occurrence of these complicated situations?

2. Road accidents constitute another source of physical disability. In Morocco, more than 80000 accidents occur every year of which 9% lead to some kind of disability. There is urgency to find ways of alleviating this disaster and their consequences. It is important to identify causative and contributing factors(drivers, road conditions, traffic signals) in order to take efficient preventive measures.

Table 4: Temporal change of the disabled population with scenario (3)

time	5	10	15	20	25	30	35	40	45	50
age 10	3.5	3.76	4.62	6.68	8.40	9.52	11.5	12.55	13.51	14.32
age 20	3.68	6.54	11.76	16.92	21.68	22.56	25.64	28.3	29.5	29.9
age 30	3.9	7.88	15.95	22.80	26.04	31.8	33.51	35.4	36.6	37.5
age 40	4.85	8.76	16.25	24.16	30.21	33.3	37.91	38.64	41.12	41.16
age 50	5.36	11.35	21.33	28.19	32.8	34.66	38.45	39.12	41.51	42.24
age 60	5.42	12.02	21.81	31.4	33.5	35.71	38.9	39.49	41.66	42.84
age 70	5.87	12.55	23.11	32.19	36.2	36.93	39.14	40.8	41.55	42.92

3. Last, but not least, in spite of the many major achievements of modern medicine, the number of congenital disabilities is still, high especially in developing countries. Improvement of medical care and sensitization of the population should be combined to reduce handicaps at birth or developed in childhood.

The results presented in this paper indicate that a lowering of both the congenital disability and the handicapping rates will result in a reduction of the proportion of the disabled population at different levels of time and age. For instance, after 50 years time, the percentage of the disabled population of age 70 may be reduced from 42% (scenario (3)) to 25% (scenario (1)). Likewise, after 20 years time, the percentage of the disabled population of age 20 may be reduced from 17% (scenario (3)) to 5.5% (scenario (2)).

These suggestions are in accordance with the St. Vincent Declaration issued in 1989 by the representatives of health departments, diabetes experts and organizations from all European countries [15]. In the case of diabetes complications, the declaration stipulated a reduction by one third limb amputations, new blindness, end-stage renal failure and morbidity from coronary heart disease. Developing countries in general, and Morocco in particular, are also urged to take actions in the spirit of the former declararation and the World Health Organization programme "Health for All". There is a twofold benefit in the use of our proposed model with simulations by national health organizations and health-care policy makers:

(a) To manage the needs of a given disabled population. The model predicts the age structure of the disabled population and consequently allows decisions to be made for each age category with regards to education, health, employment, and other needs.

(b) To control, as far as possible, the proportion of the disabled population in different age categories by making efficient decisions to help reduce the rate of congenital disability and other predictable controllable forms of disability.

Conclusion

The disabled population constitutes a class of people needing special care and necessitating important economic and social effort. Rather than dealing with the management of the disability per say, our purpose is to suggest efficient methods to prevent disability as much as possible. The mathematical model proposed illustrates clearly the dynamics of the disabled population in each age category and for different periods of time. In order to be able to compare different scenarios, a quantification of the handicapped population is obtained through numerical simulation with variable values of the parameters. Medical strategies may be considered on the basis of this model. For instance, congenital disability may and should be reduced by awareness of the ability to control pregnancy and childbirth, information and sensitization can reduce the excessive number of accidents leading to disability, and finally, diabetes education and other health care measures can reduce the incidence of disabilities throughout the entire age spectrum.

Tables 2, 3 and 4 illustrate clearly the evolution of the disabled population with time and age according to different scenarios. It can be seen that a reduction of the rate of congenital disability and/or the handicapping rate will induce a reduction of the number of disabled people in all age categories. The results of the simulations using our model can help health-care planners predict the actual needs of each age category with regards to education, employment, housing, etc. Moreover, the proportion of the disabled population can certainly be reduced if health authorities adopt a strategy that helps lower the congenital disability and the handicapping rates.

Authors' contributions

A.B contributed to the formulation of the model, numerical analysis and English writing. A.C contributed to the bibliography, parameters estimation, discussion and TEX writing.

All authors read and approved the final manuscript.

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